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 **Argument-Oriented Math Questioning**

**Overview**

The Common Core Standards for math include 8 Standards for Mathematical Practices (SMPs). Several of them require that students communicate the thinking that they undertake to solve problems. SMP 1 (‘Make sense of problems and persevere in solving them’) includes: Students must be able to ‘explain their thought processes,’ and ‘discuss, explain, and demonstrate solving a problem with multiple representations and in multiple ways.’ SMP 5 (‘Use appropriate tools strategically’) requires students to ‘explain their reasoning for their selection [from a variety of tools for solving a problem].’ And SMP 6 (‘Attend to precision’) demands that students ‘communicate their reasoning and solution to others.’

Where the new national math standards most explicitly embrace argumentation, though, is in SMP 3, ‘Construct viable arguments and critique the reasoning of others.’ The description embedded in this standard is worth quoting at length, to expatiate on the argumentative move that education leaders recognize that more rigorous 6th – 12th grade mathematics instructional standards have made.

Mathematically proficient students . . . justify their conclusions, communicate them to others, and respond to the arguments of others. Mathematically proficient students are able to compare the effectiveness of two plausible arguments, distinguish correct reasoning from that which is flawed, and if there is a flaw in an argument, explain what it is.

Many math education experts believe that more rigorous, evolved mathematics standards must include teacher questioning, and facilitated student interrogation of each other: structured discussion that elicits and promotes student mathematical reasoning, justification of solutions, and argumentation about methods and processes. Dr. Phil Daro, of the University of California Berkeley, and an author of the Common Core Standards for mathematics, testifies in the PBS Learning Media documentary ‘Making the Case,’ that the overall objective of the CCS math standards is for instruction to place a much greater emphasis on mathematical reasoning.

**Using These Argument-Oriented Questions**

These argument-oriented sets of questions – for use by the instructor in a 6th – 12th grade classroom, or by students with their peers, facilitated by the instructor – are designed to promote the kind of mathematical proficiency in which students can justify their conclusions, explain their reasoning, make arguments about their mathematical processes, critique the reasoning and arguments of others, and evaluate competing math claims and approaches.

Each category of question asks for a different angle in on SMP 3 and the other standards that include explaining and arguing for one’s mathematical reasoning. Questions within each category vary in their level of difficulty. Effective instruction will use the multiple categories to achieve proper differentiation – appropriate to the kind of math problems being studied, and to student ability and readiness to engage productively with the question.

**Argument-Oriented Questioning Exercise**

The focus of this particular application of the argument-oriented questions is to have students justify their conclusions, and then have other students critique those justifications and explanations. This argument/counter-argument structure should be followed by an opportunity for both sides (but particularly for the student who makes the first argumentative explanation) to evaluate the clashing reasoning in a manner that ultimately validates their side.

The Argument-Oriented Math Questioning Flow Sheet is an essential instrument that the instructor should use to track this mathematical argumentation. There are four individual examples of this argumentative clash ‘flowable’ on each flow sheet. Additionally, students’ participation in this mathematical argumentation exercise can be tracked on the A.C.E. In-Class Argumentation Tracking Form, using either a rating (1 – 5) or a √+, √, or √-, in one or more of the Key Components categories.

The instructor should have a set of problems that students can do individually or in groups. When completed, the instructor should assign some number (depending on the amount of class time available) of students who will **justify** their mathematical reasoning, and a group of students who will **critique** mathematical reasoning. Note that ‘critique’ doesn’t mean exclusively ‘contradict.’ ‘Critique’ can mean agree but with a difference, a variation, or a slight difference in explanation. The important thing is that there are competing ideas in play.

**Additional Suggestions**

There is a widespread consensus among math education experts that it is important that teachers pause after posing questions. These spaces or silences can be uncomfortable at first (‘Why does no one know the answer?’ ‘Where is the evidence of learning, or even of attentiveness?’ are thoughts shared by teacher and student alike). But they actually project the instructor’s understanding that the question is a challenging one, requiring careful thought. The pauses come to be seen as a gesture of an instructor’s self-confidence and respect for the process she has established.

It is also a mathematics education ‘best practice’ to plan what kind of questions will be asked and what kind of responses (a) students are likely to give to them, and (b) would be ideal. It is also important to anticipate wrong answers, and low-level responsiveness. Envisioning these scenarios in advance enables the instructor to plan for an instructionally productive argument-oriented discussion.

**Reflect on and Explain Mathematical Reasoning**

* How do you approach or solve a similar problem, such as \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ ?
* Could you use your process of solving this problem to another problem (or another set of numbers), such as \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ ?
* Is your mathematical reasoning true for all similar problems? Are there any exceptions that you can identify where your reasoning would not apply?
* Walk us through your steps. Where did you begin, and how did you get to the next step? Then the step after that . . . . ?
* Can you explain that in a different way?
* What broader principle or rule does your reasoning seem to suggest is generally true in math?

**Justify Mathematical Process**

* Why did you decide to use this method (or process) of solving the problem?
* Can you explain your mathematical process to the class? Clarify for everyone why it works.
* Is there a more efficient strategy (or process) to solve this problem?
* Do you think that your process would work with other numbers (or another problem), such as \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ ?
* What if you had started with \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ rather than \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_? Would your process still work? Why or why not?
* I’m going to re-state your justification in a somewhat different way, and you tell me if your justification and mine are in agreement. [The re-phrasing can be intentionally in agreement or intentionally not in agreement.]
* Has this same process been used in other problems that we have worked on this year?

**Argue for Your Solution**

* Why is your solution true?
* If someone were to question your solution or your approach to the problem, what might they focus on? Why would they be wrong if they made this kind of counter-argument?
* What would you say is the second best approach to solving this problem, and why is it second to the approach you chose?
* Would your conclusion still be true if we the problem were changed, in this way: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ ? Why or why not?
* What if you had started with \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ rather than \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_? Would your process still work? Why or why not?

**Critique Mathematical Reasoning**

* Can you summarize in your own words the mathematical reasoning \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ just described?
* What do you think about what \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ said?
* Do you agree with the reasoning offered by \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ to justify their answer or argue for their mathematical processes?
* Does anyone think that there is some aspect of that mathematical reasoning that is unnecessary/untrue/inapplicable/unjustified?
* Does anyone have the same answer by a different way to justify it (or argue for their mathematical process)?
* Can you describe another method with which to approach this problem or another process by which to solve this problem that also comes up with the correct answer? Now, can you argue that this alternative is better than the one that we heard from \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_?
* Would this reasoning be clear to a younger student or a person your age relatively new to math? What portion of the argument or reasoning might they not understand or not agree with, if any?
* I’m going to re-state the argumentative justification we got from \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ in a somewhat different way, and I want someone to tell me if which justification they believe is truer and why. [The re-phrasing can be intentionally better or intentionally flawed or unclear.]