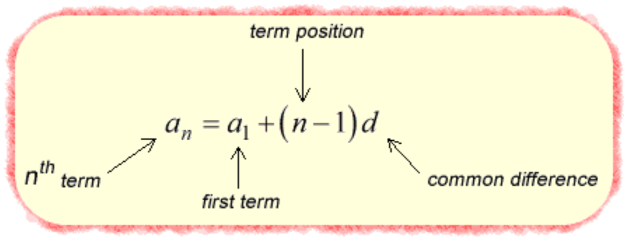


Algebraic Number Sequences and Series and Types of Argumentative Reasoning

Mathematics and Argument

Mathematics education has over the past 15 years increasingly elevated argumentation. The National Council of the Teachers of Mathematics established five 'process standards' for mathematical proficiency, and two of them (40%) are closely connected to argument: 'reasoning and proof' and 'communications.' As part of mathematical 'reasoning and proof,' students must be able to:

- 'Develop and evaluate mathematical arguments and proofs'
- 'Select and use various types of reasoning and proofs'

As part of mathematical 'communications,' students must be able to:

- 'Analyze and evaluate the mathematical thinking and strategies of others'
- 'Explanations should include mathematical arguments and rationales, not just procedural descriptions or summaries'

The Common Core Standards took mathematical argument – 'a line of reasoning that intends to show or explain why a mathematical result is true,' according to the Encyclopedia of Mathematics Education (Springer Media, 2014) – a step further when in their Standards of Math Practice 3 they require that students demonstrate a proficient ability to 'construct viable arguments and critique the reasoning of others.'

Deductive v. Inductive Argument

The algebraic number sequences and series unit gives us an opportunity to investigate differences in the basic 'types of reasoning' recognized by authorities in argument education. One fundamental dichotomy in the field separates ***deductive arguments*** and ***inductive arguments.***

According to *Argumentation: Understanding and Shaping Arguments* (Strata Publ., 2007), by Hope College professor of communication James Herrick, '***Deductive arguments*** are arguments that lead to necessary conclusions when their reasons are true.' Their 'reasons' are often called 'premises,' and they typically move top down, from the abstract to the particular: from a general or major premise coupled with a more specific observation (sometimes called a 'minor premise'), to a conclusion which, if the premises are true, also must be true.

Syllogisms are a prominent three-termed, three-step form of deductive argumentation. They lay out a major premise, then a minor premise, followed at the end by a necessary conclusion. The most famous of all syllogisms is the one from the ancient Greek philosophers who originated the form:

All men are mortal

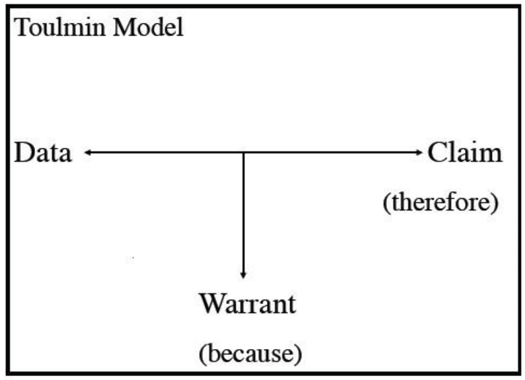
Socrates is a man

Therefore, Socrates is mortal

Logicians (i.e., those who study and write about logic) have proliferated the categories of syllogisms to a total of 24 valid types, applying variant terms like 'not' and 'some' (e.g., 'Reptiles are not warm-blooded, snakes are reptiles, therefore snakes are not warm-blooded' and 'No candy has protein, some of the food on the table is candy, therefore some of the food on the table has no protein.').

The other major category, contrasting with deductive, is the ***inductive argument.*** Inductive arguments have reasons that lead to probable but not certain conclusions. They move from the bottom up, from the particular to the more general. According to philosopher Anthony Weston, in his *A Rulebook for Arguments* (Hackett Publ., 2009), 'In [inductive] arguments, the conclusion unavoidably goes beyond the premises – that's the very point of arguing by example, authority, and so on – whereas the conclusion of a valid deductive argument only makes explicit what is already contained in the premises.'

The most famous name in inductive argumentation over the past 50 years is probably Stephen Toulmin. His *The Uses of Argument* (Cambridge University Press, 1958) laid out the highly influential inductive argument model of Claim – Data – Warrant (Argument-Centered Education typically uses the terms 'evidence' instead of 'data' and 'reasoning' instead of 'warrant' – though Toulmin's original terms are still common, the C-E-R nomenclature sounds to most students less arcane, while naming the very same thing as C-D-W).



In this model of inductive argument, 'evidence' is made up of a data set, facts, examples analogies, textual references, research, appeals to authority – objective, individual 'reasons' that the argumentative claim is likely to be true. 'Reasoning' (what Toulmin calls the 'warrant') is the analytic, rational explanation that shows how the evidence 'proves' (or strongly supports) the argumentative claim. Here's an example of an inductive argument.

Global climate change is being caused by man-made carbon emissions. According to the U.S. National Aeronautics and Space Administration (on its website, cached 11/1/15), 'Ninety-seven percent of climate scientists agree that climate-warming trends over the past century are very likely due to human activities.' The 'human activities' the scientists refer to are carbon production – the burning of coal to create energy, and the use of oil and gasoline to fuel cars, for example. Even though there can never be absolute certainty when identifying causes in nature for something as complex as the changing of global climate patterns, when a consensus of scientific experts is as overwhelming as it is on CO2 emissions' impact on climate change, we have no choice but to accept it and act on it as a scientifically verified fact.

In this example of an inductive argument:

The red text is the argumentative claim.
The green text is the evidence.
The blue text is the reasoning.

Application to Algebraic Number Sequence and Series

Now let's examine the ways that this fundamental divide in argumentation – that between deductive and inductive arguments – can be applied to mathematics generally, and algebraic number sequences and series in particular.

In instances where students are asked to choose between more than one method of solving a problem – for example, in a simple multiplication problem they might be asked to choose a method (repeated addition, long multiplication, or lattice) – and then to make an argument to justify their choice of method, their arguments will be **inductive**. 'Justification' arguments like these are generally probabilistic, not absolute. A student can supply reasons to justify their choice of method (and they can articulate those reasons in the form of evidence/reasoning, if asked to), but they are unlikely going to be able to deduce a logically impermeable conclusion from a set of abstract premises. Some of the arguments students are asked to make in math class should be inductive arguments, centered on open and arguable questions.

But many of the arguments students should be asked to make in mathematics build from deductive reasoning. They mostly are not formatted as syllogisms – with a single major premise, a single minor promise, and a necessary conclusion – but they are often deductive arguments of another kind, because they start from abstract laws, rules, or theorems; they then are given some set of numbers or equations that have to be manipulated, processed, or analyzed in relation to the premises; and finally a conclusion is to be produced, one whose validity should be an unerringly logical outcome of the preceding deductive process. If students are asked to make an argument for their answers to the problems in Chapter 5, 'Series and Sequences,' in the algebra textbook *Mathematics for the International Student* (Haese & Harris, 2012), most of the time they will be making deductive arguments.

On the other hand, it is likely that students are using a form of inductive reasoning when they identify patterns in a number sequence, or the sum of a number series, without using the general term arithmetic or geometric formula.

Arithmetic Sequence Formula

$$U_N = U_1 + (n-1)d$$

Geometric Sequence Formula

$$U_N = U_1 R^{N-1}$$

Arithmetic Series Formula

$$S_N = N/2\,(U_1 + U_N)$$

Geometric Series Formula

$$S_N = U_1 (R^N - 1)/R - 1$$

And for some problems, students are likely using a combination of inductive and deductive reasoning to obtain their solutions. Their argumentation to justify and defend their answer in these instances would similarly bring to bear both inductive and deductive argument.

ACE
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Argumentation Exercises

For each of the following problems from Chapter 5, 'Series and Sequences,' in *Mathematics for the International Student*, students should (a) solve the problem, (b) make an argument justifying their answer, and (c) identifying whether they used inductive or deductive reasoning, or both (if they use both, they should identify which portion of their argument uses which type of reasoning).

These exercises can be done with students in groups. Each student should complete the assignment individually, but then students should discuss their answers in groups, focusing especially on instances where they used a different type of reasoning. Groups can then share-out their answers and the content of their discussion where they disagreed.

Model A

Find the next two terms of: 5, 20, 80, 320 . . .

Argumentative claim
The next two terms of this sequence are 1,280, and 5,120.

Argumentative evidence/reasoning justifying the claim (i.e., answer)
The ratio of the first two terms is a : 4a. The ratio of the second and third terms is b : 4b. The ratio of the third and fourth terms is c : 4c. From this sequence over several terms, one can infer that the continuing geometric ratio for each subsequent term is 4x.

Type of argument
This is an inductive argument because it is built up from data points to a general conclusion.

Model B

Consider the arithmetic sequence 6, 17, 28, 39, 50 . . . . Find its 50^{th} term.

Argumentative claim

The 50^{th} term of this arithmetic sequence is 545.

Argumentative evidence/reasoning justifying the claim (i.e., answer)

The common difference is derivable by identifying and comparing the difference between terms 1 and 2, 2 and 3, 3 and 4, and 4 and 5. The difference common to all of these is 11. From these examples one can infer that the common difference is 11. Then, using the formula for arithmetic sequences –

$$\mathbf{U_N = U_1 + (n-1)d}$$

-- we know that:

$$50^{th}\ \text{Term} = 6 + (50 - 1)11$$

$$50^{th}\ \text{Term} = 6 + (49)11$$

$$50^{th}\ \text{Term} = 6 + 539$$

$$50^{th}\ \text{Term} = 545$$

Type of argument
The portion of the argument that infers the common difference is inductive. The rest of the argument is deductive, since it begins with the formula of solving arithmetic sequences and applies that formula to a specific set of terms, resulting in a valid conclusion.

Problems

1. Find the next two terms of: 1, 16, 81, 256 . . .

Argumentative claim

Argumentative evidence/reasoning

Type of argument

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2. Consider the arithmetic sequence: 87, 83, 79, 75 . . . . Which term of the sequence is -297?

Argumentative claim

Argumentative evidence/reasoning

Type of argument

3. Find the next two terms of: 1, 4, 9 . . .

Argumentative claim

Argumentative evidence/reasoning

Type of argument

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4. Find the general term U_N for an arithmetic sequence with $U_7 = 41$ and $U_{13} = 77$.

Argumentative claim

Argumentative evidence/reasoning

Type of argument

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5. Valeria joins a social networking site. After 1 week she has 34 friends. After 2 weeks she has 41 friends, after 3 weeks she has 48 friends, and after 4 weeks she has 55 friends. After how many weeks will Valeria have 150 friends?

Argumentative claim

Argumentative evidence/reasoning

Type of argument

6. Consider the geometric sequence: 12, -6, 3, ... Find the general term U_N.

Argumentative claim

Argumentative evidence/reasoning

Type of argument

7. A herd of 32 deer is to be left unchecked on a large island off the coast of Alaska. It is estimated that the size of the herd will increase each year by 18%. How long will it take for the herd size to reach 5000.

Argumentative claim

Argumentative evidence/reasoning

Type of argument

8. A soccer stadium has 25 sections of seating. Each section has 44 rows of seats, with 22 seats in the first row, 23 seats in the second row, 24 in the third row, and so on. How many seats are there in the whole stadium?

Argumentative claim

Argumentative evidence/reasoning

Type of argument

9. **Find the sum of the following series to 10 terms: 5 + 10 + 20 + 40 . . . .**

Argumentative claim

Argumentative evidence/reasoning

Type of argument

10. **A geometric sequence has first term 6 and common ration 1.5. The sum of the first *n* terms of the sequence is 79.125. Find *n*.**

Argumentative claim

Argumentative evidence/reasoning

Type of argument

Extension

Read the Theory of Knowledge section on pp. 145 – 146 of *Mathematics for the International Student* and (using the back of this sheet, if necessary) answer these three questions, using the terms and concepts *deductive* and *inductive arguments* and *reasoning.*

1. Can we prove that a statement is true in all cases by checking that it is true for some specific cases?

2. How do we know when we have proven a statement to be true?

3. Is it reasonable for a mathematician to assume a conjecture is true if it appears true in multiple cases but cannot be or has not been proven formally?

In groups complete the accompanying Sudoku exercise. Reflect on the type of reasoning you used to approach and solve the problem. Discuss with your group-mates.

4. Did you use inductive reasoning, deductive reasoning, or some combination of both to solve the problem?

5. Which form of reasoning works best to solve Sudoku problems like this one, based on your discussion within your group? On what basis do you come to this conclusion?